

## RECTANGULAR DISK 3 dB HYBRIDS

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## Abstract

Four novel types of rectangular disk 3 dB hybrids have been obtained through the constituting principle on the basis of field distribution of two dipole modes, (1,0) and (0,1) modes. Their hybrid properties, however, actually are spoiled by the impairing influence of the higher order modes with increasing of center frequency. Therefore, some methods of improvement on them to a practicable level also have been described. Experimental results verify the improvement methodology.

## Introduction

Recently, microwave components, such as hybrids, power dividers, and filters, have been newly designed by means of planar circuit approach [1]~[3]. These components are marked by simple shape of usefulness for applications at high microwave frequencies. In particular, a circular disk 3-dB hybrid has been investigated in detail, and some methods of designing for favorable characteristics have been reported [4],[5].

This paper treats a 3-dB hybrid consisting of a rectangular disk resonator with two pairs of ports symmetrically arranged. This hybrid possesses some features different from a circular disk hybrid, e.g., port-arrangements parallel or perpendicular to one another, four various types of circuit configurations, and so forth, though the constituting principle is similar to each other.

First, after explaining the constituting principle on the basis of electromagnetic field distribution of only two dipole modes (fundamental (1,0) and (0,1) modes) in the rectangular disk, we show that this hybrid is classified into four types. Next, we try to improve on the characteristics spoiled by the impairing influence of higher order resonant modes nearby the fundamental resonant frequency. Finally, we obtain experimental corroborations by measuring the scattering parameters of some typical hybrids.

In this paper, triplate-type circuits are dealt with. In connection with experiment, moreover, the relative permittivity of the spacing material and the spacing between the center conductor and the ground conductor are 2.53 and 1.53 mm, respectively.

## Constituting principle and circuit configurations

In this section, we consider the circuit illustrated in Fig. 1. Let it be assumed that the rectangular disk is square ( $a=b$ ) and only the dipole modes are excited

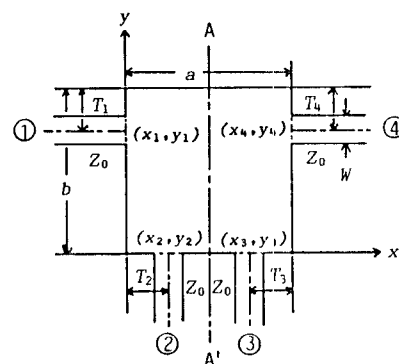


Fig. 1. Schematic diagram of rectangular disk circuit with four coupling-ports. This circuit becomes fundamental configuration of a type-A hybrid when  $a=b$  and  $T_1=T_2=T_3=T_4$ .

by a signal input through port 1. Then, the voltage in the square distributes as follows:

$$V(x, y) = V_0 \{ \cos(\pi x/a) - \cos(\pi T/a) \cos(\pi y/a) \} \quad (1)$$

where  $T \equiv T_1 (= T_2 = T_3 = T_4)$ .

Hence, a voltage at each port  $V_i (i=1\sim 4)$  can be written as

$$V_1 = V_0 \{ 1 + \cos^2(\pi T/a) \}, \quad V_2 = 0 \quad (2a, b)$$

$$V_3 = -2V_0 \cos(\pi T/a), \quad V_4 = -V_0 \sin^2(\pi T/a) \quad (2c, d)$$

From Eqs. (2a), (2c), and (2d) the following equation is derived:

$$|V_1|^2 = |V_3|^2 + |V_4|^2 \quad (3)$$

This equation states that port 1 is matched if the ports 3 and 4 are terminated in an impedance equal to that of port 1. Moreover, Eqs. (2) shows that ports 1 and 2 are isolated to each other, and the relative amplitudes of ports 3 and 4 are related by

$$|V_3|/|V_4| = 2\cos(\pi T/a)/\sin^2(\pi T/a) \quad (4)$$

Now, if we choose a power-split ratio equal to 1, we have

$$T = 0.364a \quad (5)$$

A similar discussion is possible in case of excitation from the other ports also. Therefore, we can expect that this coupler becomes a 3-dB hybrid with isolations between ports 1 and 2, and between ports 3 and 4.

Similarly, the other three types of circuit configurations shown in Fig. 2 also are anticipated as 3-dB hybrids. In the following, however, we will mainly give the examinations of type-A hybrids out of space consideration.

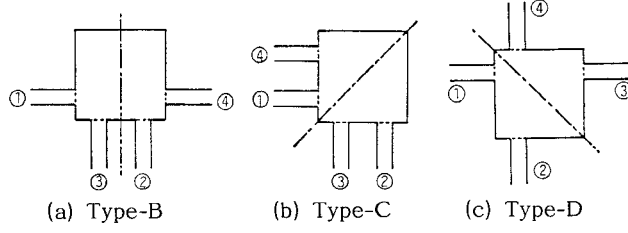
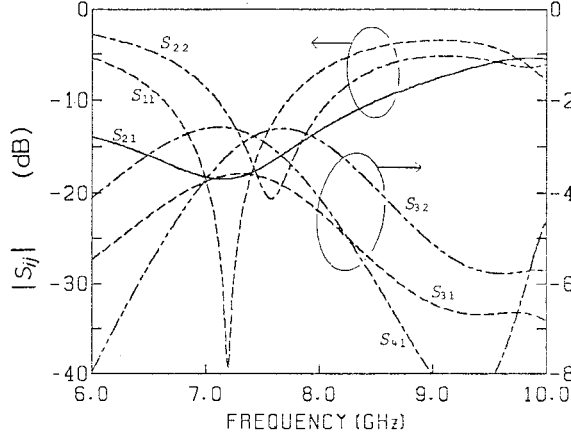


Fig. 2. Schematic diagrams of other types.



Design parameters:  $a=13.47\text{mm}$  ( $f_{10}=f_{01}=7\text{GHz}$ )

$T_1=T_2=T_3=T_4=4.90\text{mm}$ ,  $Z_0=50.0\Omega$

Fig. 3. Computed values of  $|S_{ij}|$  vs. frequency for a fundamental type-A hybrid.

### Scattering matrices

The impedance-matrix elements of the circuit shown in Fig. 1 can be obtained from the Green's function of the second kind for the rectangular segment as follows:

$$Z_{ij} = j \frac{\omega \mu d}{2ab} \sum_m \sum_n \frac{\epsilon_m^2 \epsilon_n^2}{(k_{xm}^2 + k_{yn}^2 - k^2)} \cos(k_{xm} x_i) \cos(k_{yn} y_i) \cdot \cos(k_{xm} x_j) \cos(k_{yn} y_j) \frac{\sin(p_i W/2) \sin(p_j W/2)}{(p_i W/2)(p_j W/2)} \quad (6)$$

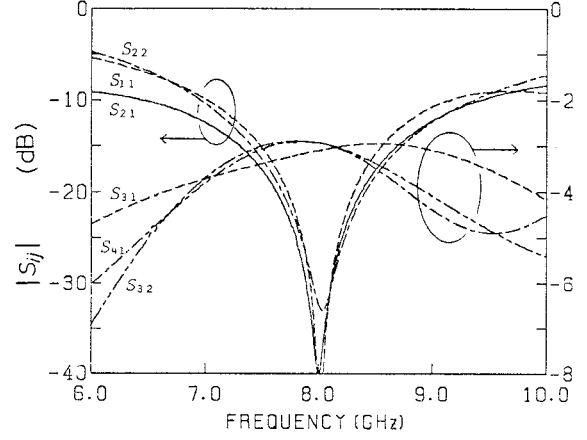
where

$$k_{xm} = m\pi/a, \quad k_{yn} = n\pi/b, \quad k^2 = \omega^2 \epsilon \mu,$$

$$p_{i,j} = \begin{cases} k_{xm} & (y_{i,j}=0, b) \\ k_{yn} & (x_{i,j}=0, a) \end{cases}, \quad \epsilon_m = \begin{cases} 1 & (m=0) \\ \sqrt{2} & (m \neq 0) \end{cases}, \quad \epsilon_n = \begin{cases} 1 & (n=0) \\ \sqrt{2} & (n \neq 0) \end{cases}.$$

The more familiar scattering parameters are then obtained by using standard Z-matrix to S-matrix transformation.

The magnitudes of the S-matrix elements as a function of frequency for a fundamental type-A hybrid with  $a=13.47\text{mm}$  (resulting in the fundamental frequency  $f_{10,01}=7\text{GHz}$ ) and  $T_1=T_2=T_3=T_4=4.90\text{mm}$  were computed. This fundamental circuit, as can be seen from Fig. 3,



Design parameters:  $a=12.11\text{mm}$ ,  $b=14.44\text{mm}$

$T_1=3.66\text{mm}$ ,  $T_2=3.41\text{mm}$

Fig. 4. Computed scattering parameters of an improved type-A hybrid with rearranged ports and modified size of rectangular disk.

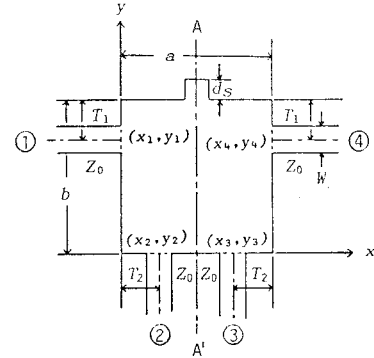


Fig. 5. Configuration of a type-A hybrid with a capacitive stub.

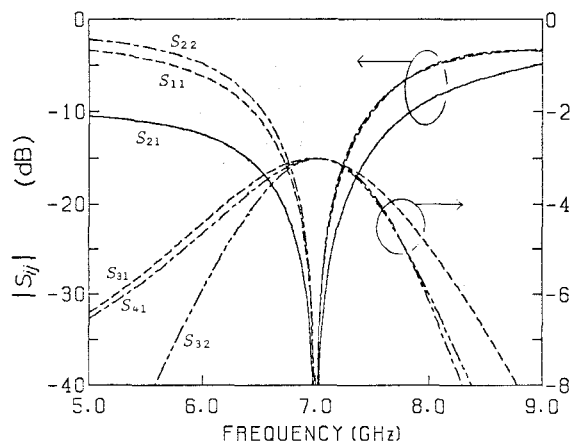
suffers considerably from a deterioration in hybrid property by the debasing influence of the higher order resonant modes.

### Improvements on hybrid characteristics

In this section, we try to improve on the spoilt characteristics upon consideration of the higher order modes and without loss in simplicity of the structure. Powell's method [6] is employed as a mathematical technique for optimization.

First, the positions of the coupling-ports and the size of the rectangular disk are determined by the above optimization technique. As an example, the computed S-parameters for an improved type-A hybrid are shown in Fig. 4. Its optimized design parameters are given as the insert in the figure also.

A better improvement can be obtained by adding an appropriate convex (capacitive stub) at the circum-



Design parameters;  $a=13.92\text{mm}$ ,  $b=13.61\text{mm}$

$T_1=3.15\text{mm}$ ,  $T_2=5.05\text{mm}$ ,  $d_S=2.95\text{mm}$

Fig. 6. Computed scattering parameters of the improved type-A hybrid with a capacitive stub.

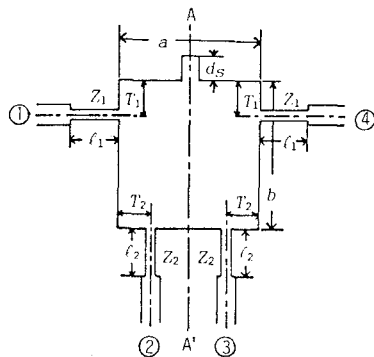
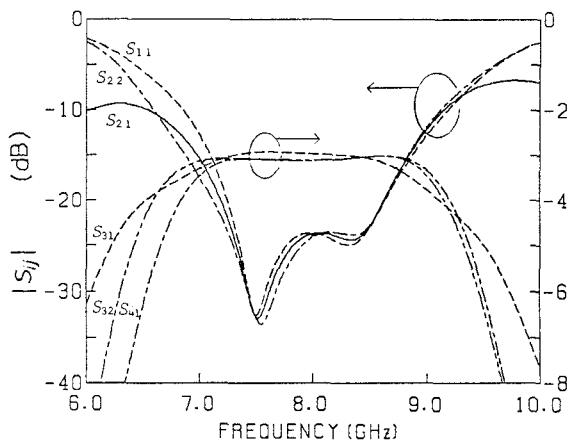


Fig. 7. A circuit pattern of a type-A hybrid with a capacitive stub and impedance steps.

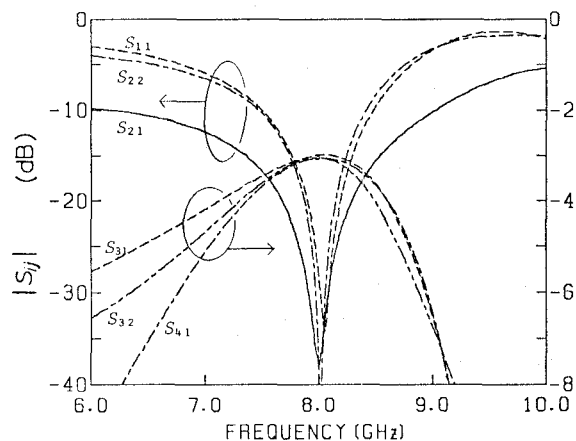
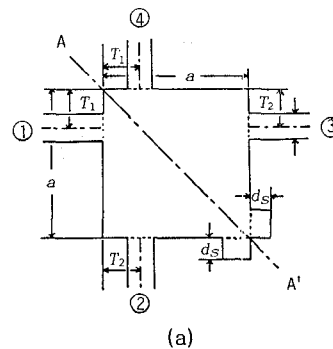


Design parameters;  $a=11.78\text{mm}$ ,  $b=12.19\text{mm}$

$T_1=1.40\text{mm}$ ,  $T_2=3.58\text{mm}$ ,  $Z_1=90.7\Omega$ ,  $Z_2=91.1\Omega$

$\ell_1=9.92\text{mm}$ ,  $\ell_2=10.92\text{mm}$ ,  $d_S=2.83\text{mm}$

Fig. 8. Computed scattering parameters of the improved type-A hybrid with high impedance-steps and a capacitive stub.



Design parameters;  $a=11.80\text{mm}$ ,  $T_1=3.97\text{mm}$

$T_2=5.65\text{mm}$ ,  $d_S=0.84\text{mm}$

(b)

Fig. 9. (a) A circuit pattern of a type-D hybrid with two capacitive stubs and (b) its computed scattering parameters.

ference of the disk as shown in Fig. 5. Fig. 6 shows the optimum design parameters and the computed  $S$ -parameters. Here, however, the center frequency is chosen 7 GHz, because ports 2 and 3 overlap each other for the center frequency of 8 GHz.

Next, we try to widen the bandwidth by inserting an impedance step between the rectangular disk and each coupling-port in the manner illustrated in Fig. 7. As can be seen from Fig. 8, the bandwidth of a type-A hybrid with high-impedance steps is considerably broadened by using jointly a stub.

Similar improvements are obtained for other types also, but here omitted with the exception of a type-D hybrid with two additional capacitive stubs shown in Fig. 9.

### Experimental results

In order to confirm the above computed results, some tested hybrids were fabricated on a 1/16-inch-thick Rexolite 1422. Fig. 10, 11, and 12 exhibit the X-

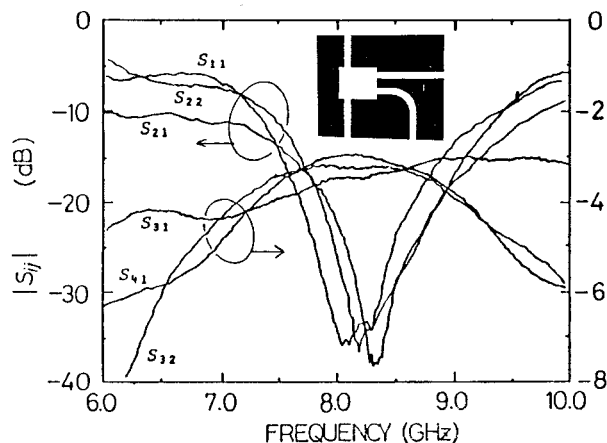


Fig. 10. Measured scattering parameters of the experimental type-A hybrid with rearranged ports and modified size of rectangular disk (Refer to Fig. 4).

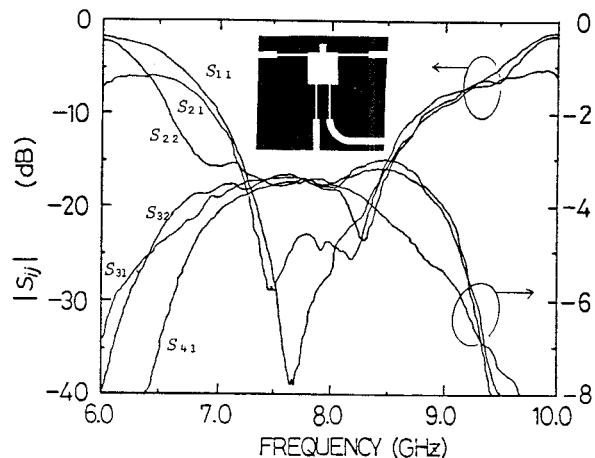


Fig. 11. Measured scattering parameters of the experimental type-A hybrid with a capacitive stub and high impedance-steps (Refer to Fig. 8).

Y recordings of  $|S_{ij}|$  made with HP's 8755S Frequency Response Test Set corresponding to the hybrids shown in Fig. 4, 8, and 9, respectively. The theoretical and experimental results are in good agreement with each other.

### Conclusions

A constituting principle of rectangular disk 3 dB hybrids and methods of improving the characteristics to a practicable level have been described. As four different types occurs in principle, we can properly use them according to the requirement for the layout on a circuit. An examination of a rectangular disk directional coupler with an arbitrary power-split ratio would be an interesting subject for further work.

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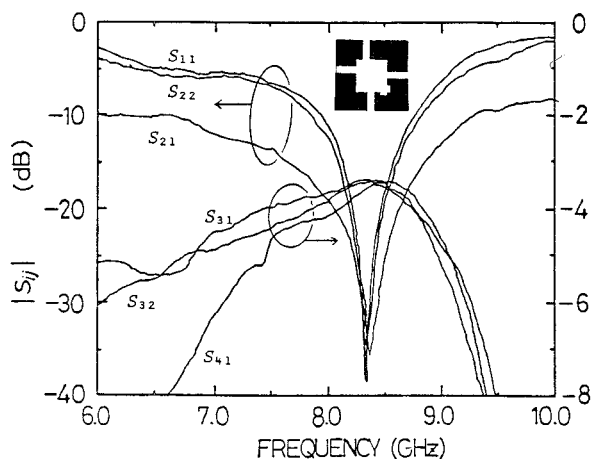


Fig. 12. Measured scattering parameters of the experimental type-D hybrid with two capacitive stubs (Refer to Fig. 9).

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